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Instructors' Perceptions of their Students' Conceptions: The Case in Undergraduate Mathematics

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How a student conceives the nature of a subject they study affects the approach they take to that study and ultimately their learning outcome. This conception is shaped by prior experience with the subject and has a lasting impact on the student's learning. For subsequent education to be effective, an instructor must link the current topic to the student's prior knowledge. Short of assessing their students, an instructor relies on their subjective experience, intuitions, and perceptions about this prior knowledge. These perceptions shape the educational experience. The current study explores, in the context of undergraduate mathematics, the alignment of instructors' perceptions of student conceptions of mathematics and the students' actual conceptions. Using a version of the Conceptions of Mathematics Questionnaire, instructors of lower-year courses were found to have overestimated, while upper-year course instructors underestimated, their students' fragmented conceptions of mathematics. Instructors across all years underestimate their students' cohesive conceptions. This misalignment of perspectives may have profound implications for practice, some of which are discussed.

It is now well established that the perceptions a student has of a subject they study affects their approach to studying, and ultimately their performance in, that subject (Biggs & Tang, 2011; Trigwell & Prosser, 1991). A deeper, connected view of the subject correlates to a deeper approach to study and better outcomes, both in terms of quantitative performance (e.g., assessment scores) and conceptual gains (Trigwell & Prosser, 1991). Fragmented, superficial perspectives often result in less desirable outcomes. Given this evidence on the impact of a student's perspective of a subject on their performance in that subject, a key to improving student performance may be in fostering shifts in their perceptions. That is, students may come to view a subject more cohesively if the learning situations they experience emphasize the cohesive structure of the subject. A major barrier to implementing this shift may lie with the instructors. Do instructors actually know how their students view their subject? An exploration of this question in the context of undergraduate mathematics is the topic of this study.

Fragmented conceptions of a subject include viewing the subject as a disjointed collection of facts and/or operations (Crawford, Gordon, Nicholas, & Prosser, 1994; Crawford, Gordon, Nicholas, & Prosser, 1998a; Crawford, Gordon, Nicholas, & Prosser, 1998b). These facts and/or operations can be applied to solve problems, but a larger, complete picture is lacking. Students who hold fragmented conceptions of a subject learn topics in isolation and generally lack connections between these topics. A cohesive conception sees the facts as interrelated, comprising a consistent and logical totality. Applications still remain, and a cohesive view allows the student to draw on a richer set of tools for use with these applications.

In terms of mathematics, the subject considered in the present study, fragmented and cohesive conceptions, have for some time played a central role in the mathematics education discourse. Fragmented conceptions of mathematics are closely linked to the *instrumental understanding* of Skemp (1976) and the *procedural knowledge* of Hiebert and Lefevre (1986). With this type of understanding a student knows that a procedure, for example, is appropriate given the context but is not necessarily able to apply the procedure efficiently or flexibly. The procedure is for the student an isolated and rigid construct. For example, a student may be able to solve a system of equations consistently with a certain algorithm but not understand the algorithm deeply enough to modify it for use in a given situation (Star, 2005). Cohesive conceptions resemble Skemp's (1976) *relational understanding* and Hiebert and Lefevre's (1986) *conceptual knowledge*. This level of understanding involves a richer experience of mathematics. Students with this level of understanding comprehend why a procedure is appropriate for a given context and are able to tailor the procedure to make it more efficient. These students are also able to draw upon a number of procedures, perhaps innovating their own, and decide upon which is most appropriate.

Of course, a subject like mathematics comprises both procedures and concepts, and a university mathematics curriculum requires students to be proficient in both. How these two constructs interact and develop in a student's mind is still a matter of debate, but it is generally agreed upon that solid conceptual knowledge facilitates procedural knowledge more easily than the reverse. The most current research suggests that both are best developed in an iterative process, with gains in procedural knowledge balanced with gains in conceptual knowledge, and vice versa (Rittle-Johnson & Schneider, 2014). However, if students view mathematics as a disjointed collection of procedures and facts—that is, if they have a fragmented view of mathematics—without regard to the greater conceptual structure of mathematics, this balancing of procedures and concepts may be a difficult task.

How students view a subject also affects their approach to learning that subject. Students who hold a fragmented view of a subject tend to adopt surficial approaches to study, focusing on memorization and the acquisition of facts and procedures for immediate use. The act of study for such students is geared toward the completion of tasks, involves lower-level skills, such as memorization, and seldom involves longer-term retention (Biggs & Tang, 2011). Students with a cohesive view, on the other hand, are more likely to take a deep approach to study, focusing on understanding and seeing the subject as a connected whole; see (Prosser & Trigwell, 1999) for a review of the early literature and (Biggs & Tang, 2011) for an updated review. These approaches to study translate into different learning outcomes (Biggs, 1979; Marton & Säljö, 1976). Deep approaches have been found to correlate with higher course grades—though not always (Campbell & Cabrera, 2014; Choy, O'Grady, & Rotgans, 2012; Trigwell & Prosser, 1991)—and greater conceptual gains, while surficial approaches often result in less desirable outcomes (Watkins, 2001; Zeegers, 2001).

In this current study, students and their instructors were given a survey designed to measure their conceptions of mathematics. While the students were asked to complete it as truthfully as possible, the instructors were asked first to reflect on their current class and form an image of their "archetypal" or "average" student and then to complete the survey as they think this archetypal student would. The intention with this exercise was to quantify a practice commonly done by mathematics instructors. Anecdotally—though, also see (Engelbrecht, Harding, & Potgieter, 2005)—instructors often refer to their students using statements such as, "My students do not understand this concept," or, "They think of math as just pushing numbers around." These perceptions may be partially informed by responses by students on assessments, but they also comprise instructor perception bias. The educational experiences offered by the instructors are, in turn, shaped by these perspectives of their students. A companion study (Maciejewski & Merchant, 2015) evaluates the relationship between the questionnaire scores reported here, study approaches taken by the students, and resulting course grade.

The results of this study indicate a divide between how instructors perceive their students' view the nature of mathematics and how the students actually view mathematics. The direction of this divide, whether instructors over or underestimate aspects of their students' conceptions, is dependent upon the level of the course being taught by the instructor.

Methods

Participants

An email invitation to participate in the current study was circulated in the second regular semester of the 2013/2014 school year to all members of the mathematics department of a major Canadian research university who were currently teaching a course. In total, 23 instructors responded and volunteered to participate. These instructors also agreed to have the students of one of their current courses, as some instructors were teaching more than one course, contacted and invited to participate. All students in the 23 classes were sent email invitations and 322 students across the 23 courses volunteered to participate. A random draw for four gift cards for campus student businesses was used as an incentive.

Student participation by course varied, from four in the sole fourth-year course to 23 in a second-year course. On average the participation rate by course was roughly 15%. However, this study concerns students and instructors grouped by course year. The numbers for this partitioning are in Table 1. Since there was only one fourth-year course, and since this course had only four study participants, the course was grouped with the third-year courses to create the third/fourth-year category. A comparison between the mean course grade of each course sample with that of the entire course revealed no systematic sample bias (results not reported). Therefore, there is no evidence to suggest the samples are not representative.

Measures

The students and instructors completed a version of the Conceptions of Mathematics Questionnaire (CMQ) (Crawford et al., 1998a). The CMQ used in this study and the preambles given to the students and instructors are found in the Appendix. The CMQ gives scores to a participant on two scales that correspond to fragmented and cohesive conceptions of mathematics. Fragmented conceptions comprise viewing mathematics as essentially a computational system and a body of factual knowledge. Cohesive conceptions involve viewing mathematics as a system of logic inspired by, and useful in, solving authentic problems. Facts and procedures are still present, and a cohesive conception views these as facets of a totality.

These two scales derive from a phenomenographic study in which students responded to the question, "Think about the maths you've done so far. What do you think mathematics is?" (Crawford et al., 1994). Two themes emerged. Some students described mathematics as the study of numbers and their applications in other disciplines. Views like these were

Table 1
Number of Students and Instructors/Courses by Course Year

Year	Number of Students	Number of Instructors/Courses
1^{st}	169	11
2^{nd}	100	6
$3^{rd}/4^{th}$	53	6

classified as fragmented conceptions of mathematics. Those with cohesive conceptions tended to describe mathematics as a logical or abstract system that is applicable to the study of the physical world, but also as a system that itself can be studied. These survey responses were used to generate the CMQ (Crawford et al., 1998). Since the questionnaire's initial publication it has been used with, and validated for, a variety of different populations (Alkhateeb, 2001; Liston & O'Donoghue, 2009; Macbean, 2004; Mji, 1999; Mji, 2003; Mji & Alkhateeb, 2005; Mji & Klaas, 2001). The initial publication on the CMQ (Crawford et al., 1998a) reports excellent internal consistency, in terms of Cronbach's alpha, for both fragmented ($\alpha = 0.85$, post-test) and cohesive ($\alpha = 0.88$, post-test) scales, which has been confirmed in the subsequent publications cited previously.

The fragmented and cohesive scales are not mutually exclusive, though reported as such in at least one study (Mji, 2003). Some of the statements in the CMQ that correspond to a fragmented conception may be agreed with by someone who holds a strongly cohesive conception of mathematics. This is not an inconsistency. Indeed, an applied mathematician may agree that mathematics is "...about formulae and applying them to everyday life and situations," (fragmented) while simultaneously agreeing that "[m]ath is a logical system which helps to explain the world around us" (cohesive). Or, perhaps less apparent, a number theorist may agree that "[f]or me, math is the study of numbers," (fragmented) and that "[m]ath is like a universal language which allows people to communicate and understand the universe" (cohesive). As Crawford and colleagues (1994) identify, a cohesive conception of mathematics encompasses aspects of fragmented conceptions, such as mathematics as procedures, though the scope of these aspects is wider and is a part of a greater connected whole for one who holds a cohesive conception of mathematics.

Analysis of Data

The CMQ survey responses for both instructors and students were first analyzed separately to verify underlying factors and validity. Since the CMQ has not previously been used with a demographic comparable to the current one, a principal component analysis with varimax rotation was performed for both the student and instructor data, and the results are reported in Table 2. The aggregate student data confirms the factor structure first reported in Crawford and colleagues (1998b). The student data was subsequently broken down into first, second, and third/fourth year sets, and analyses on these data reveal the same factor structure for these subsets of the sample (results are not reported). As was found in Crawford and colleagues (1998b), item 15 was revealed to be inconsistent and was dropped from further analyses.

The analysis of the instructor survey responses also reveals the expected factor structure; see Table 2. Though the sample was much smaller ($n = 23$) than typically recommended sizes for such an analysis—recommendations that can vary widely (Mundfrom, Shaw, & Ke, 2005)—the loadings on the two factors are quite favorable (de Winter, Dodou, & Wieringa, 2009). Many of the large positive covariances loading on one factor were matched with large negative covariances loading on the other factor. However, some of the variables are worthy of examination: item 4 loads only weakly on factor 1, and item 6 is somewhat inconsistent. Both were retained in subsequent analysis, with item 4 being attributed to factor 1 and item 6 attributed to factor 2. Also, item 15 was revealed to load on factor 1 and have a negative covariance with factor 2, a result originally anticipated by Crawford and colleagues (1998b). Item 15 was dropped from further analyses to correspond to the student survey data.

A test of internal consistency using Cronbach's alpha was also performed (Cronbach, 1951). Results are reported in Table 3. Both scales for both student and instructor samples show strong internal consistency. Considering comparisons are made between subsets of these samples determined by course year, further reliability analyses were performed on these subsets. The results are in Table 4. As is shown, good to excellent reliability exists for both students and instructors in the three given year categories.

Having confirmed the factor structure and reliability of the two samples, comparisons are made between the year subsets. Figure 1 presents the mean student and instructor CMQ scores for both the fragmented and cohesive scales, and Table 5 reports the difference in means of instructor and student CMQ scores. Note that a positive value indicates the

Table 2
Student and Instructor Conceptions of Mathematics Questionnaire Factor Analysis

		Students		Instructors	
	Items	Factor 1	Factor 2	Factor 1	Factor 2
Fragmented Items	Q1	0.71	-0.13	0.88	0.01
	Q2	0.57	-0.06	1.27	-0.34
	Q4	0.52	-0.34	0.24	0.05
	Q5	0.72	-0.11	0.79	-0.42
	Q7	0.63	0.11	0.87	-0.33
	Q9	0.70	0.06	0.93	-0.30
	Q12	0.53	0.11	0.64	-0.38
	Q13	0.73	0.08	0.99	-0.49
	Q16	0.65	0.08	1.13	-0.34
	Q18	0.62	0.11	0.60	-0.25
Cohesive Items	Q3	-0.05	0.40	-0.60	0.69
	Q6	0.07	0.55	0.32	0.81
	Q8	-0.04	0.66	-0.34	0.54
	Q10	0.08	0.73	-0.20	0.62
	Q11	0.01	0.72	-0.16	0.57
	Q14	0.11	0.50	-0.17	0.56
	Q15	0.56	0.32	0.58	-0.21
	Q17	0.03	0.54	-0.24	0.66

Note. Covariances reported

Table 3
Conceptions of Mathematics Scale Items and Internal Consistency

	Cronbach's alpha	
Scale and Representative Item	Students	Instructor
Fragmented Mathematics is about playing around with numbers and working out numerical problems.	0.85	0.94
Cohesive Mathematics is a theoretical framework describing reality with the aim of helping us understand the world.	0.83	0.85

Table 4
Conceptions of Mathematics Internal Consistency by Course Year

	Cronbach's alpha			
	Student		Instructor	
Year	Fragmented	Cohesive	Fragmented	Cohesive
1	0.80	0.85	0.90	0.88
2	0.85	0.83	0.72	0.82
3 / 4	0.90	0.75	0.90	0.75

instructor mean was greater than the student mean while a negative value indicates the instructor mean was less than the student mean. Welch's t-Tests (Welch, 1947) were conducted on the differences between means, and the resulting p values are reported in the Table 5. It was found that the mean fragmented score for the instructors (First Year (FY): M = 3.75, SD = 0.63; Second Year (SY): M = 3.45, SD = 0.34) was higher than the mean fragmented score for the students (FY: M = 3.52, SD = 0.54; SY: M = 3.13, SD = 0.64) in the first two years; not statistically significant for the first year, but significant for second year, $t(7) = 2.05$, p

Figure 1

Average Fragmented and Cohesive CMQ Scores for Instructors (Circles) and Students (Squares)

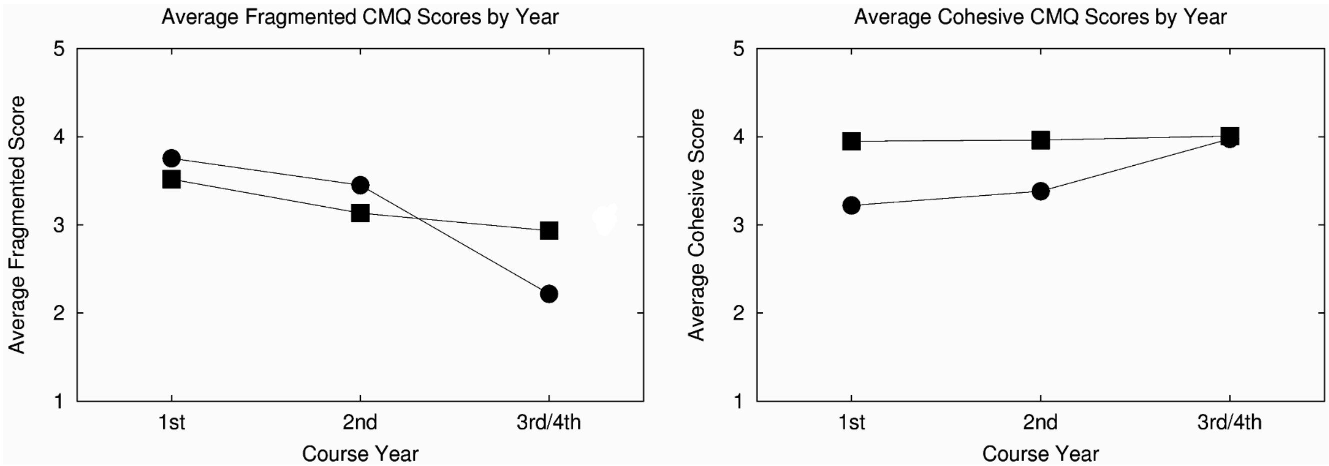

Table 5

Difference Between Instructor and Student Mean CMQ Scores

	Difference in Mean Scores		Significance (p=___)		Effect Size (d=____)	
Year	Fragmented	Cohesive	Fragmented	Cohesive	Fragmented	Cohesive
1	0.24	-0.73	0.11	0.002	0.43	-1.23
2	0.32	-0.58	0.04	0.02	0.50	-1.03
3 / 4	-0.72	-0.03	0.05	0.56	-0.89	-0.07

Note. A positive (resp. negative) difference indicates the instructor mean was greater (less) than the student mean.

= 0.04. This result is reversed in the third- and fourth-year group. There the instructors' mean fragmented score (M = 2.22, SD = 0.89) is significantly less than the students' mean fragmented score (M = 2.93, SD = 0.80), $t(6) = -1.89$, $p = 0.05$. In all years the instructors' mean cohesive score is less than the students' mean cohesive score, very significantly for the first two years ($t(11) = -3.76$, $p < 0.01$ and $t(6) = -2.74$, $p = 0.02$, respectively), but not significant for the third and fourth years.

An effect size analysis was performed using Cohen's d (Cohen, 1988) to understand better the relative differences in the means. These values are reported in Table 5. The effect size for the differences in mean fragmented conception scores in the first two years are moderate (FY: $d = 0.43$; SY: $d = 0.50$) and large for the final two years, $d = 0.89$. For the differences in the mean cohesive conception scores, the effect is large in the first two years and practically nil in the last two.

Since there is such a marked difference in the instructors' perspectives in the first two and the last two years, it is worthwhile to evaluate if there is a similar difference in the students' conceptions. Table 6 reports the differences in student conceptions between years. There is a very significant negative difference in mean fragmented score between first and second year, $t(181) = -5.00$, $p < 0.01$, and a somewhat significant negative difference in mean fragmented score between second and third/fourth year, $t(88) = -1.56$, $p = 0.06$. There are slight positive differences in mean cohesive scores, but neither of these differences is significant.

Summary of Results

When asked to complete the conceptions of mathematics questionnaire as they think their archetypal student would, instructors in the first two years score, on average, significantly higher on the fragmented scale and significantly lower on the cohesive scale than their students. Instructors in the last two years score, on average, significantly lower on the fragmented scale and somewhat lower on the cohesive scale than their students.

There is a marked difference between first/second year and third/fourth year instructors' fragmented and cohesive scores. This suggests the possibility that there is a significant difference between how instructors of lower and upper-year courses perceive their students' conceptions of mathematics.

Table 6
Difference in Student Average CMQ Scores Between Years

Year	Fragmented			Cohesive		
	Average	Difference	Significance (p=___)	Average	Difference	Significance (p=___)
1	3.52	N/A	N/A	3.95	N/A	N/A
2	3.13	-0.34	p ≈ 0	3.96	0.01	.57
3 / 4	2.93	-0.20	0.06	4.00	0.05	0.70

Students, on average, have greater fragmented conceptions of mathematics in the first two years than in the last two, but they are fairly consistent in their cohesive views across all years. This contrasts with their instructors' difference in perspective.

Discussion

This study has found that university math instructors may perceive their students as conceiving mathematics differently than what they actually do. Lower-year instructors perceive their students to have greater fragmented conceptions and much lower cohesive conceptions, while upper-year instructors perceive their students to have much less fragmented conceptions. Essentially, there is a clear divide between how instructors of early year and later year courses think their students view mathematics. This stands in contrast to how the students actually view mathematics. First year students hold much higher fragmented conceptions than later, third/fourth-year students—which is expected, as many of the first year courses are "service" courses taken by students in programs where math is otherwise not a major component. These first-year courses are, for many students, terminal in that they are the extent of university mathematics these students will experience. But even though there is a prominence of fragmented conceptions in the earlier years, instructors overestimate how prominent these conceptions are. Though these conceptions are lower in the later years, upper-year instructors underestimate how widely held they actually are. Instructors in all years underestimate their students' cohesive conceptions of mathematics, albeit less so in upper years. Perhaps what makes the perceptual difference between early- and later-year instructors even more profound is that the students present essentially the same cohesive views of mathematics across all undergraduate years. That is, the instructors' perceptual differences do not correspond to a difference presented by the students.

How the current work may be used to inform practice remains to be seen. It is likely that an instructors' perception of their students, including how they view the subject, informs what experiences the instructor provides the students. This may, in turn, make for tasks and assessments that conflict with how the students view the subject. For example, if an instructor believes their students hold fragmented, procedure-oriented conceptions of mathematics, they may think the students are not prepared for a conceptually-oriented task. This may be a missed opportunity, and such a disconnect can have profound implications for student development. When learning tasks are aligned with the skills and perspectives brought by the students, all students are capable of taking a deeper approach to learning (Biggs, 1999; Biggs & Tang, 2011).

It is well established that a component of effective education involves activating students' prior knowledge. The most successful education connects all new experiences to students' prior knowledge (Ambrose, Bridges, DiPietro, Lovett, & Norman, 2010; Ausubel, Novak, & Hanesian, 1978; Resnick, 1983). If an instructor's perception of their students' prior knowledge does not align with their actual prior knowledge, then this connection cannot be made. As Ambrose and colleagues (2010) identify, "...it is critical to assess the amount and nature of students' prior knowledge so that we can design our instruction appropriately." As it stands, it is not a common practice for instructors to assess their students' prior knowledge. Without such an assessment, an instructor is left to make assumptions about the composition and nature of students' prior knowledge. These assumptions may not be accurate, creating a disconnect between what is to be learned and what has been learned.

In university introductory mathematics courses, instructors are currently witnessing dramatic year-to-year differences in the prior mathematical experiences brought with students entering from high school. Primary and secondary math education focuses more and more on conceptual aspects of mathematics and downplays algorithms and calculations. These experiences shape how students view the subject. The shift in focus to concepts in primary and secondary school necessitates a corresponding shift to concepts in introductory university-level mathematics courses, which are currently often procedure-heavy service calculus courses. Without such a shift, the transition from high school to the university—well documented as a chasm between university expectations and student abilities (De Guzman, Hodgson, Robert, & Villani, 1998)—will be all the more difficult, and student outcomes are likely to decline. Despite this need, first-year mathematics courses have remained largely static in their content and delivery over the last few decades. This disconnect between first-year instructors' expectations and entering students' abilities is exasperated by instructors' inaccurate perceptions of their students' views of mathematics (Engelbrecht et al., 2005). Instructors think there is a match between the procedure-heavy first-year curriculum and their, perceived to be, procedurally-minded students. Students who are less procedurally-minded under-perform in these courses, causing instructors to think of their students as having impoverished procedures. The instructor in a subsequent iteration of the course incorporates this experience by focusing further on procedures. All along, the focus is on fragmented conceptions of mathematics when it ought to be on cohesive conceptions.

This disconnect may not be unique to the high school/university transition. The results of this study indicate that a similar disconnect appears between the lower and upper years of the university. For mathematics there is a tangible difference between lower and upper year courses. Lower year courses are often service courses, and this is reflected in the curricula through an emphasis on procedures and applications. Few upper year courses are intended as service courses, and the curricula are more concept-focused. The ways these two types of curricula are enacted also differs substantially. Tasks and assessments given to first-year students typically involve solving large numbers of short, procedure-based problems. In upper-year courses the students are most commonly assessed on their understanding of theorems and how they might be applied. It is perhaps this difference in course emphasis that leads instructors to view their students differently.

The marked divide between lower- and upper-year instructors' perceptions is especially surprising given that upper-year students were once lower year students. Granted, a good portion of the students that hold fragmented conceptions leave the mathematics course streams after the first year to pursue their non-mathematics-oriented specializations. But, nonetheless, the underestimation by upper-year instructors of fragmented conceptions held by their students seems to suggest that instructors may assume the students that continue in mathematics are undergoing a shift in their conceptions of mathematics in their first two years. The data reported here indicates that such a shift may not be actually occurring. Indeed, procedure-heavy service courses may only serve to reinforce students' fragmented conceptions.

Of course, the above claims, though likely, need to be substantiated. Students' perspectives of their instructors, learning situations, subjects, etc., have all been extensively studied (Prosser & Trigwell, 1999). Instructors' perceptions of their students, on the other hand, seems to be an almost entirely unexplored domain. It is a potentially interesting and insightful domain, given the results of the current study.

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